Research Article

HOW DIAGRAMS CAN IMPROVE REASONING

Malcolm I. Bauer and P.N. Johnson-Laird

Princeton University

Abstract—*We report an experimental study on the effects of diagrams on deductive reasoning with* double disjunctions, *for example:*

Raphael is in Tacoma or Julia is in Atlanta, or both.
Julia is in Atlanta or Paul is in Philadelphia, or both.
What follows?

We confirmed that subjects find it difficult to deduce a valid conclusion, such as

Julia is in Atlanta, or both Raphael is in Tacoma and Paul is in Philadelphia.

In a preliminary study, the format of the premises was either verbal or diagrammatic, and the diagrams used icons to distinguish between inclusive and exclusive disjunctions. The diagrams had no effect on performance. In the main experiment, the diagrams made the alternative possibilities more explicit. The subjects responded faster (about 35 s) and drew many more valid conclusions (nearly 30%) from the diagrams than from the verbal premises. These results corroborate the theory of mental models and have implications for the role of diagrams in reasoning.

How can diagrams help people to reason? The question has a long philosophical history, but a short psychological one. Philosophers, for example, have worried about how the use of a single diagram in a geometric proof might mislead geometers (Beth, 1971). In a pioneering psychological article, Larkin and Simon (1987) distinguished the role of diagrams in three separate sorts of processes: search, recognition, and inference (see also Tabachneck & Simon, 1992). Larkin and Simon said diagrams can make it easier to find relevant information: One can scan from one element to another element nearby much more rapidly than one might be able to find the equivalent information in a list of numbers or verbal assertions. Diagrams can make it easier to identify instances of a concept: An iconic representation can be recognized faster than a verbal description. Their symmetries can reduce the number of cases that need to be examined. However, about inference, Larkin and Simon (1987) wrote:

> In view of the dramatic effects that alternative representations may produce on search and recognition processes, it may seem surprising that the differential effects on inference appear less strong. Inference is largely independent of representation if the information content of the two sets of inference rules [one operating on diagrams and the other operating on verbal statements] is equivalent—i.e. the two sets are isomorphs as they are in our examples. (p. 71)

Address correspondence to Malcolm I. Bauer, Department of Psychology, Princeton University, Green Hall, Princeton, NJ 08544.

Barwise and Etchemendy (1992) have argued that the truth behind the adage that a picture is worth a thousand words is that diagrams and pictures are good at presenting a wealth of specific conjunctive information. "It is much harder to use them," they said, "to present indefinite information, negative information, or disjunctive information" (p. 80). Such information is often better conveyed by sentences, and so their pedagogical program, Hyperproof, makes use of both diagrams and sentences. It appears that these researchers are skeptical about how diagrams can aid in inference, especially reasoning depending on disjunctions or negations. The present article provides a theoretical basis for why diagrams can help with such reasoning, and describes two experiments that give empirical support to our claims. The results have implications for the role of imagery in reasoning, and we comment briefly on this point as well.

A deduction is valid if its conclusion must be true given that its premises are true. Formal logic provides methods of testing validity, and nearly all psychological theories of reasoning have postulated the existence of formal rules of inference in the mind (Braine, 1978; Inhelder & Piaget, 1958; Rips, 1983; Smith, Langston, & Nisbett, 1992). An inference is difficult, according to such theories, if it calls for a long chain of steps in the derivation of its conclusion, or if it calls for a rule of inference that is difficult to access or to apply. Diagrams are unlikely to affect performance, however, for a reason similar to the one adduced by Larkin and Simon (1987): Once the logical form of the problem has been extracted from a diagram, the same chain of deductions based on the same rules of inference should unfold.

The theory of mental models tells a very different story (Johnson-Laird, 1983; Johnson-Laird & Byrne, 1991). Granted the definition of a valid deduction (see above), any method of testing validity is in effect a method of ensuring that the conclusion holds in all the possible states of affairs characterized by the premises. Instead of relying on formal rules of inference, the model theory postulates a more direct process mirroring the examination of possibilities: Individuals reason by (a) constructing a model, or models, based on the information in the premises and background knowledge, (b) formulating a conclusion that is true of the model and subject to other constraints, such as parsimony, and (c) searching for alternative models in which the conclusion does not hold. If there is no such alternative model, then the conclusion is valid. In general, deductions depending on multiple models should be difficult, and erroneous conclusions to them should be consistent with the truth of the premises, because the subjects consider some, but not all, of the possible models. These predictions, which cannot be made by any existing theories based on formal rules, have been corroborated in studies of all the main sorts of deduction (see Johnson-Laird & Byrne, 1991).

Deductive reasoning often depends on taking into account alternative possibilities, and a major source of errors is the difficulty of holding in mind several models simultaneously. A premise such as "All the women in the room are psychologists" is consistent with the existence, or not, of psychologists in the room who are not women, and with the existence, or not, of individuals in the room who are neither women nor psychologists. Many deductions, however, can be drawn without explicitly representing such possibilities. For example:

All the women in the room are psychologists.
All the psychologists in the room are Russians.
Therefore, all the women in the room are Russians.

A similar phenomenon occurs with reasoning based on conditionals (see Johnson-Laird, Byrne, & Schaeken, 1992). So, in what circumstances are reasoners forced to consider alternative possibilities? One way in which to elicit such representations is, according to the model theory, to use a disjunction, that is, a premise of the form "A or B," where A denotes one proposition and B denotes another proposition.

Much evidence exists to show that disjunctions are harder to think about than conjunctions. Osherson (1974–1976), for example, in characterizing children's and adolescent's deductive competence, observed that disjunctions are harder than conjunctions. Many studies of concept attainment bear out this claim (e.g., Bruner, Goodnow, & Austin, 1956; Neisser & Weene, 1962). Likewise, studies of deduction have also corroborated it (Braine, Reiser, & Rumain, 1984). Theories based on formal rules of inference cannot explain the phenomenon, but merely assume post hoc that rules for conjunction are easier to access or to apply than those for disjunction (Braine et al., 1984; Rips, 1983). However, the difference follows directly from the model theory because a conjunction calls for only one explicit model, whereas a disjunction calls for at least two explicit models.

If the model theory is right, then there should be a breakdown in deductive performance as the number of models increases beyond the capacity of working memory. One way in which to increase models is to use so-called double disjunctions, such as

Raphael is in Tacoma or Julia is in Atlanta, or both.
Julia is in Atlanta or Paul is in Philadelphia, or both.
What follows?

The premises support five alternative models:

t	a	p
t	a	
	a	p
	a	
t		p,

where each line represents a separate model of a possible state of affairs, "t" denotes Raphael in Tacoma, "a" denotes Julia in Atlanta, and "p" denotes Paul in Philadelphia. It is indeed difficult to draw a correct, valid conclusion, such as

Raphael is in Tacoma and Paul is in Philadelphia, or Julia is in Atlanta.

Inclusive disjunctions (in which either or both of the propositions can be true to satisfy the "or") are harder than exclusive disjunctions (in which one but not both of the propositions must be true to satisfy the "or"), because inclusive disjunctions require more models (see Johnson-Laird et al., 1992). Likewise, consider a *contrary* problem, in which one atomic proposition is contrary to another:

Raphael is in Tacoma or Julia is in Atlanta, or both.
Julia is in Seattle or Paul is in Philadelphia, or both.
What follows?

Such a problem is harder than an *identical* problem, in which the identical proposition occurs in both disjunctions (as in the first example above). The model theory assumes that an additional mental operation is required to grasp that one state of affairs rules out another.

In contrast to the remarks about diagrams cited earlier and to the formal-rule theories, the model theory predicts that certain sorts of diagrams should help reasoning. Any device that helps reasoners to keep track of alternative models of the premises should be useful. Therefore, diagrams that help reasoners to make explicit the alternative states of affairs needed for reasoning should improve performance. An obvious domain in which to test this prediction is double-disjunctive reasoning, because subjects are known to perform poorly and to have difficulty in keeping track of all the different possibilities. We carried out two studies of diagrams and disjunctive reasoning in order to test three predictions: First, diagrams can improve reasoning if they help reasoners to make explicit the alternative possibilities. Second, exclusive disjunctions should be easier than inclusive disjunctions. Third, identical double disjunctions, in which the identical atomic proposition occurs in the two disjunctions, should be easier than contrary double disjunctions, in which a proposition in one disjunction is contrary to a proposition in the other disjunction.

THE EXPERIMENTS

In a preliminary study, we used diagrams such as the one in Figure 1 to represent the double disjunctions. The two lines connecting pairs of ellipses intersect at a square, which represents inclusive disjunction. We used a circle containing a cross to represent exclusive disjunction in other problems. The experiment confirmed that exclusive disjunctions are easier than inclusive disjunctions, and that identical problems are easier than contrary problems, but diagrams (28% correct conclusions) had no effect on performance in comparison to the verbal problems (32% correct conclusions). In retrospect, we believe the diagrams failed to make sufficiently explicit the alternative states of affairs, and whether the disjunctions were exclusive or inclusive. Arbitrary symbols—the box or the circle with a cross—represented the form of disjunction. Similarly, the diagrams failed to make explicit negative instances of propositions (i.e., a particular individual need not be within the oval representing a city in order to satisfy a premise). Subjects could imagine an individual outside an oval, but the diagram itself did not make this possibility explicit. In our main experiment, accordingly, we used a different sort of diagram. The aim of the experiment was to test whether subjects would reason more

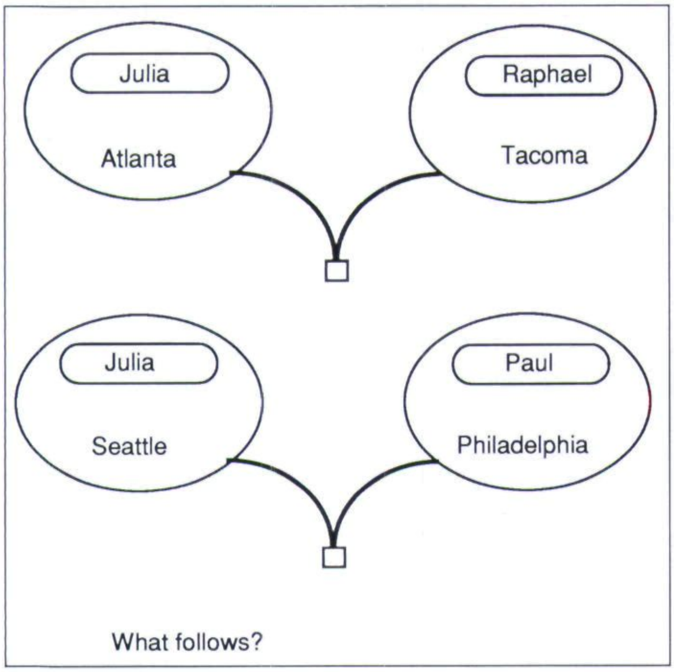

Fig. 1. Diagram representing a double disjunction (contrary inclusive) in the preliminary study.

accurately with diagrams making alternative states of affairs explicit than with verbal premises. To examine the generality of the phenomenon, we used two different domains. One concerned people and places, and the other concerned switches and lights.

Method

Design

We tested four independent groups of subjects based on two factors: whether the domain was people and places or electrical circuits, and whether the format of the problems was verbal or diagrammatic. The problems were double disjunctions: Each group tried to solve four problems based on exclusive or inclusive double disjunctions that were identical or contrary. We used Williams squares to counterbalance the order of presentation of the four problems over the subjects in each group.

Materials

The verbal presentation of the circuit problems was as follows (for a contrary inclusive disjunction). The subjects understood that A, B, and C referred to switches in the same circuit.

While the light is on:
A or B is on (or both).
B is off or C is on (or both).
Note: B can be in one of three positions: on, off, or stand by.
The light is on.
What follows?

The standby position of the switch (in which it is neither on nor off) was introduced so that the circuit problems were equivalent to the people-and-places problems. For example, Julia may be in Atlanta according to one premise, or she may be in Seattle according to the other premise, but there is a third possibility that she is in neither of the two places. The standby position of a switch is equivalent to this possibility.

To make the problems in the two domains identical in form, for the people-and-places domain, the subjects were told that the premises had to hold for a certain event to occur. The people-and-places problems were accordingly of the following sort:

While the event is occurring:
Raphael is in Tacoma or Julia is in Atlanta, or both.
Julia is in Seattle or Paul is in Philadelphia, or both.
The event is taking place.
What follows?

The diagrams were designed to help the subjects make explicit the different possibilities of exclusive and inclusive disjunctions, and of propositions and their negations. Figure 2 shows the diagram for the contrary inclusive circuit. Disjunction is represented by two switches in parallel. Closing one switch (or both in the inclusive case) allows electricity to flow through that section of the circuit. Negation is represented by yoking two switches together in opposite modes: The dotted line between switch B and switch $B_{opposed}$ yokes the switches so that both cannot be closed together, but both can be open. To represent exclusive disjunction, the circuit showed that when both switches were closed, the current ceased to flow because of a short circuit.

Figure 3 shows the diagram for the people-and-places problem of the same logical form. The subjects understood that they had to complete a path from one side of the figure to the other by inserting the shapes corresponding to people into the slots in the path corresponding to places. As the figure shows, the shape corresponding to a person can fit only into a similarly shaped slot corresponding to a place. Hence, in this case, the shape designating Julia could be in Atlanta or Seattle, or neither—just as either one of the two yoked switches in the circuit diagram could be closed, or neither. To represent exclusive

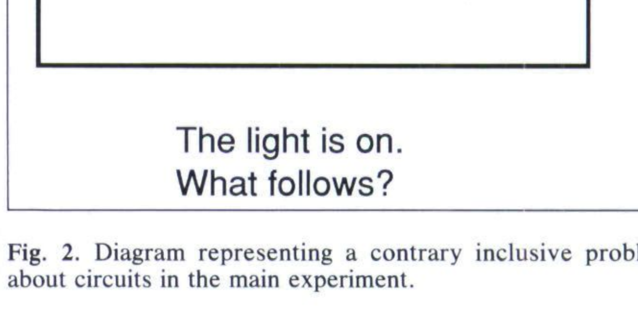

Fig. 2. Diagram representing a contrary inclusive problem about circuits in the main experiment.

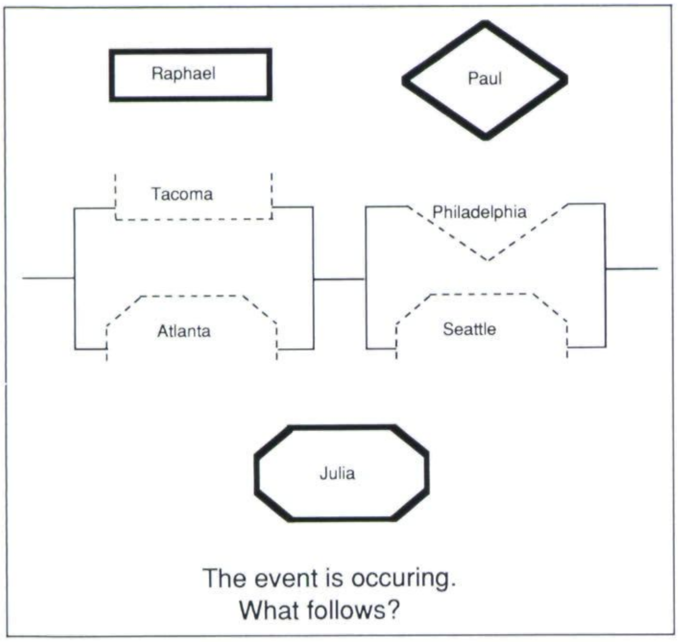

Fig. 3. Diagram representing a contrary inclusive problem about people and places in the main experiment.

disjunction, the two disjunct shapes had attachments that occupied each other's slot, so if one shape was inserted into its slot, the other was prevented from being inserted into its slot.

Subjects

We tested 48 Princeton University undergraduate volunteers, 12 in each group, and none of them had any training in formal logic or circuit analysis. They were paid \$5 for participating in the experiment, which lasted for about 35 min.

Procedure

The subjects were tested individually. They were told that they would solve a practice problem and four other problems. The task was explained in the following terms (for the verbal form of the people-and-places domain):

> In order for a certain event to occur certain people have to be in particular American cities. You will be given two rules about who is in what city. Both of these rules must be satisfied for the event to occur. You will then be given a fact about the event (whether it occurred or not). Your task is to determine what follows from the rules and the fact.

The instructions went on to explain the interpretation of the disjunctions, and to make clear that one person could not be in two places at the same time. There were comparable instructions for the circuit problems. The instructions for the diagram groups were similar, except they stated that subjects were to be given a diagram (instead of rules) and then described how the connections in diagrams should be interpreted, analogous to the meaning of disjunction in the verbal groups. Finally, the experimenter answered the subjects' questions, asked the subjects to repeat back their instructions to detect any misconceptions, and presented the practice problem. The subjects were then given the four problems to solve, one at a time.

Results

Figure 4 presents the percentages of correct conclusions for each sort of problem and format. The content of the problems, circuits or people and places, had no effect on either accuracy ($F[1, 44] = 1.0$, $p > .3$) or speed of response ($F[1, 44] = 1.0$, $p > .3$), so we have collapsed the data over these two conditions. As the figure shows, the diagrams increased accuracy in a striking way: The subjects drew 74% correct conclusions for the diagram problems in comparison with 46% correct conclusions for the verbal problems, $F(1, 44) = 14.9$, $p < .001$. The results confirmed that exclusive disjunctions are easier than inclusive disjunctions ($F[1, 44] = 24.3$, $p < .001$), and that the identical problems are easier than the contrary problems ($F[1, 44] = 4.2$, $p < .05$).

Figure 5 presents the mean response times (in seconds) for the problems. Because there were few correct responses to the inclusive disjunctions in the verbal format, we could not make a statistical test of the response times for correct conclusions alone. Instead, we analyzed the response times for all conclusions, both correct and incorrect. The subjects responded reliably faster to the diagrams (99 s) than to the verbal problems (135 s), $F(1, 44) = 8.59$, $p < .01$. The subjects responded faster to exclusive disjunctions (104 s) than to inclusive disjunctions

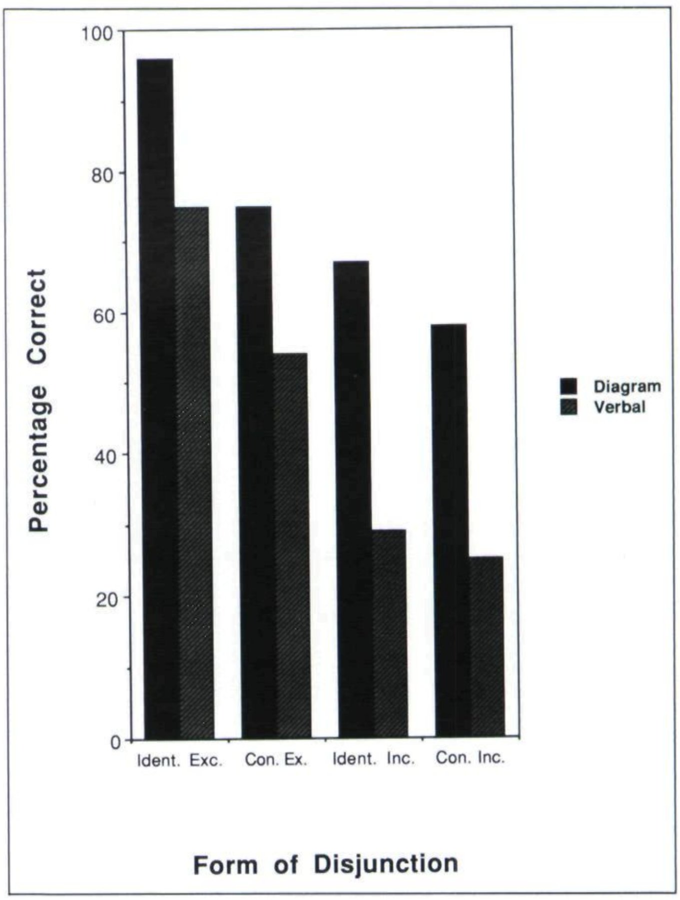

Fig. 4. Percentages of correct conclusions in the main experiment for each disjunction form (identical/contrary by inclusive/exclusive) for the diagrammatic and verbal problems.

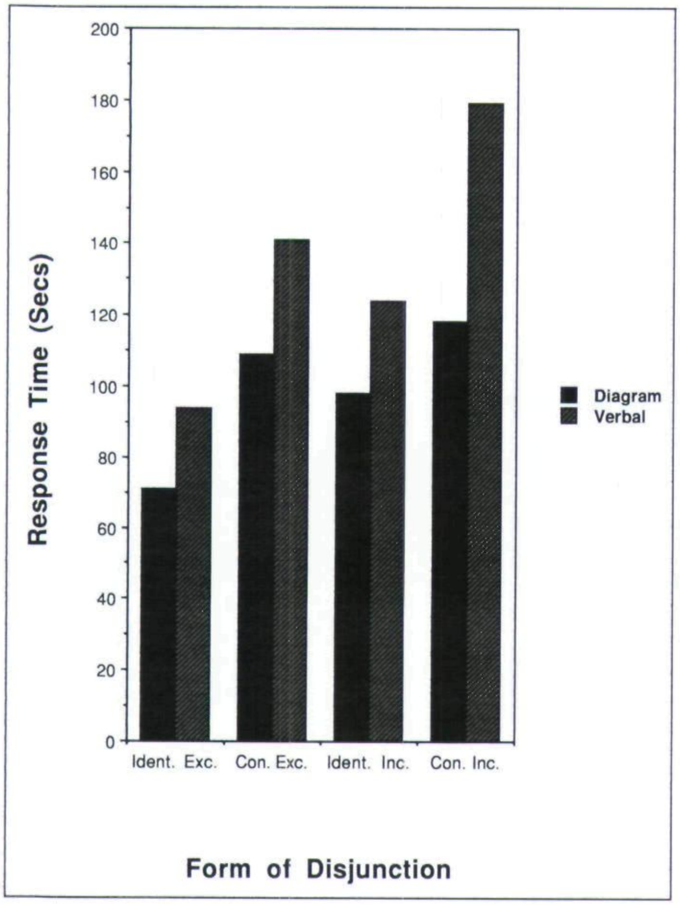

Fig. 5. Mean response times (in seconds) for conclusions in the main experiment for each disjunction form (identical/contrary by inclusive/exclusive) for the diagrammatic and verbal problems.

(130 s; $F[1, 44] = 24.3, p < .001$), and they responded faster to identical problems (97 s) than to contrary problems (137 s; $F[1, 44] = 52.2, p < .001$). Designed comparisons showed that the subjects working with diagrams drew correct conclusions from exclusive disjunctions reliably faster than from inclusive disjunctions ($F[1, 7] = 13.4, p < .01$), and that they drew correct conclusions from identical problems reliably faster than from contrary problems ($F[1, 7] = 8.5, p < .05$). In summary, the subjects were faster and more likely to make a valid inference from the diagrams than from the verbal premises.

We classified the subjects' conclusions into four broad categories: disjunctions, mixed forms, conditionals, and restatements of the premises. Disjunctive conclusions enumerate a list of alternative possibilities:

Karl is in Los Angeles and Lydia is in Detroit, or
Karl is in Los Angeles and Marcie is in Minneapolis, or
Lydia is in Baltimore and Marcie is in Minneapolis.

These conclusions correspond directly to the models postulated by the theory. The mixed conclusions contain separate major constituents that capture the possibilities:

(Karl is in Los Angeles or Lydia is in Baltimore) and Marcie is in Minneapolis, or Marcie is not in Minneapolis and Lydia is in Detroit and Karl is in Los Angeles.

The conditional conclusions are in the overall form of conditionals:

If Lydia is in Detroit, then Karl is in Los Angeles and Marcie is not in Minneapolis.

Finally, restatements are conclusions that merely restate the conclusions or make trivial surface variants of them.

The subjects always drew a conclusion, and the percentages of the different sorts were as follows: 65% disjunctions, 18% mixed forms, 12% conditionals, and 5% restatements. In all conditions, disjunctions were more frequent than other kinds of conclusions (Wilcoxon's $T = 274, p < .01$). The verbal presentation of circuit problems yielded fewer disjunctive conclusions and a concomitant increase in the other forms of conclusion (Mann-Whitney's $U = 89.0, p < .002$). It seems that the verbal presentation of a circuit problem is difficult, because it leads to fewer complete enumerations of the possibilities and to more conclusions that merely restate the premises.

The model theory predicts that subjects who err should tend to do so by constructing some, but not all, of the models of the premises, and so errors should be consistent with the premises. This prediction was corroborated by our results: Seventy percent of the erroneous conclusions were consistent with the premises, and thus only 30% were inconsistent with the premises (Wilcoxon's $T = 25.5, p < .0001$). Because diagrams should make the alternatives more readily available, we can predict that the proportion of inconsistent errors should be smaller with them than with verbal problems. The results bear out this prediction. The total numbers of errors were as follows:

Consistent errors with diagrams:	22
Inconsistent errors with diagrams:	3
Consistent errors with verbal problems:	31
Inconsistent errors with verbal problems:	20

The proportions of inconsistent errors were in the predicted direction, but, excluding subjects who made no errors, the difference was not reliable (Mann-Whitney's $U = 125.5, p = .12$).

If subjects sometimes guess or make superficial linguistic manipulations of the premises, then their answers might well be inconsistent with the models of the premises. Superficial linguistic manipulations are less likely to yield conclusions of the disjunctive form, but they might yield mixed or conditional conclusions. Hence, errors that are not consistent with the premises should be more likely to occur with mixed or conditional conclusions than with disjunctive conclusions. This prediction was corroborated: For disjunctions, errors tended to be consistent with the premises (34%) rather than inconsistent (6%), whereas for the mixed and conditional conclusions, errors tended to be inconsistent with the premises (28%) rather than consistent (18%). This interaction was reliable (Wilcoxon's $T = 49.5, p < .0001$).

We also found a figural effect in the subjects' conclusions to the problems (see Johnson-Laird & Byrne, 1991, chap. 6, for a full description of the figural effect). In problems such as

"Raphael is in Tacoma or Julia is in Atlanta, or both. Julia is in Atlanta or Paul is in Philadelphia, or both. What follows?" subjects had an overwhelming bias to draw conclusions that mentioned the first atomic expression (Raphael is in Tacoma) prior to the last one (Paul is in Philadelphia). Ninety-two percent of the conclusions were of this form ($p = .5^{24}$). Interestingly, the effect was very strong for the diagram condition. Out of 12 subjects, 11 showed the figural bias in the diagram problems, and there was one tie ($p = .5^{11}$). This is strong evidence that the figural effect is not due to the linguistic form of the premises.

GENERAL DISCUSSION

In our preliminary study of double disjunctions, we confirmed that exclusive disjunctions are easier than inclusive disjunctions, and that disjunctions containing identical propositions are easier than disjunctions containing contrary propositions. Diagrams such as the one in Figure 1 had no significant effect on improving performance. In our main experiment, however, diagrams such as those in Figures 2 and 3 produced a massive improvement in the percentage of valid conclusions that the subjects drew—30% more than in the case of problems stated verbally. In reasoning with these diagrams, the subjects were also reliably faster—by about 35 s—than in the verbal conditions. The percentages of accurate conclusions also confirmed that exclusive disjunctions are easier than inclusive disjunctions, and that identical problems are easier than contrary problems. Unlike the studies in Johnson-Laird et al. (1992), the experimental procedure yielded a measure of processing time, and the results of both the preliminary and the main experiments showed for the first time that subjects are some 13 to 26 s faster to draw conclusions based on exclusive disjunctions than on inclusive disjunctions, and that they are faster to draw conclusions from identical disjunctions than from contrary disjunctions.

Why were the diagrams in our main experiment so helpful? Unlike those in the preliminary study, they did not depend on an arbitrary icon to express disjunction or to distinguish between the two sorts of disjunction. The disjunctive alternatives were laid out topographically as switches in parallel or alternative routes from one side of the diagram to the other, and the contrast between inclusive and exclusive disjunctions was depicted by whether or not it was physically possible to close both switches or to insert the pieces that would complete both routes. The diagrams were static, but subjects could readily envisage the alternative ways of completing the circuit by closing a switch, or completing the path by moving this piece or that into its appropriate slot. They could also readily envisage the completion of one part of a path as opposed to leaving it incomplete (the contrast between an affirmative and a negative proposition). The diagrams, in fact, showed the negative state of affairs, but it was easy to imagine closing the switches, or sliding the pieces into their appropriate slots, to represent the corresponding affirmative propositions. In short, the diagrams helped the subjects to envisage the alternative possibilities inherent in the premises: Disjunction was translated into a spatial analog—alternative routes or circuits—and affirmation versus negation was translated into an easily envisaged operation on mental representations of the diagrams.

The results corroborate the theory of mental models in three distinct ways. First, unlike theories based on formal rules, the theory predicts that diagrams can help people to reason. The problem in reasoning, according to the theory, is to keep track of alternative possibilities, and so devices such as diagrams that help reasoners to make these possibilities explicit should improve reasoning. Second, the theory predicts that exclusive disjunctions should be easier than inclusive disjunctions, because exclusive disjunctions require fewer models than inclusive disjunctions. Existing formal-rule theories can accommodate this difference, but they cannot predict it: They can assess the difficulty of the rules of inference for the two sorts of disjunction only post hoc. The model theory also predicts the difference between identical and contrary problems, though it is possible that this prediction can be made by certain theories based on formal rules. Our results bear out these predictions in terms of accuracy, and for the first time they show that it takes longer for individuals to reason with an increased number of models. Third, errors characteristically were consistent with the premises, as the model theory predicts if subjects overlook some possible models of the premises. As one might expect, diagrams had a tendency, albeit a not quite significant one, to reduce the proportion of errors that were inconsistent with the premises. There were only three such errors with diagrams.

The verbal form of the conclusions also suggested that model-based reasoning minimizes errors that are inconsistent with the premises. The typical form of a conclusion was a disjunctive enumeration of the possible states of affairs implied by the premises, as one would expect if the subjects were trying to build a set of the alternative models of the premises. A greater proportion of errors inconsistent with the premises occurred with other sorts of conclusions, which may have been derived by superficial linguistic manipulations of the premises. Neither the form of the conclusions nor the form of the errors can be readily explained by theories based on formal rules (Braine, 1978; Rips, 1983; Smith et al., 1992). These theories do not systematically generate invalid conclusions, because they are based on valid rules of inference. Also, the existence of the figural effect in the diagram condition strongly supports the notion that the figural effect is not due to linguistic form but is a more general phenomenon due to the order in which information enters working memory.

Individuals evidently attempt to construct models of all the different situations compatible with the premises. In the case of verbal premises, this task calls for the recovery of the meanings of the premises, followed by the construction of models from these meanings. The process is taxing on the capacity of working memory, and it is all too easy to lose track of which particular situations have been represented by models. In the case of the diagrammatic problems, the subjects form a visual representation of the diagram, and in their mind's eye they can imagine moving the pieces or switches (i.e., they carry out visual transformations of images). Bypassing the construction of the meanings of verbal premises and manipulating visual images appear to reduce the load on working memory and to speed up the process of inference. As a result, reasoners are much less likely to overlook possible configurations, and so they tend to draw more accurate conclusions. This supports the notion that diagrams are not merely encoded in propositional representations equivalent to those constructed from verbal premises—in contrast to the claims of researchers who subscribe to propositional representation theories (e.g., Baylor, 1971; Palmer, 1975; Pylyshyn, 1973). Imagery is in no way epiphenomenal in this task: Subjects manipulate their images of the diagrams in order to reach their conclusions. If the diagrams were translated into an underlying propositional representation, there would be no way to explain the improved performance when diagrams were presented. The verbal premises would presumably be translated into similar, or even simpler, propositional representations.

Readers may be tempted to suppose that double disjunctions are an unimportant and unrepresentative form of reasoning. In fact, they merely exacerbate a characteristic difficulty in reasoning: They explicitly increase the number of possibilities that reasoners have to bear in mind. Other sorts of reasoning, such as syllogisms, conditional inferences, and inferences with multiple quantifiers, are also more difficult as the number of possible models of the premises increases. Indeed, whenever there are disjunctive possibilities, cognitive tasks become more difficult—whether in concept attainment (Bruner et al., 1956), in decision making (Shafir & Tversky, in press; Tversky & Shafir, in press), or deduction itself (Johnson-Laird & Byrne, 1991). The cognitive causes of disasters in everyday problem solving are likewise often attributable to a failure to consider disjunctive possibilities. For example, the operators at Three Mile Island inferred that the high temperature at a relief valve was caused by a leak and overlooked the possibility that the valve was stuck open. The same difficulties are likely to arise in constructing and evaluating inferences based on informal reasoning or argumentation (Voss, Perkins, & Segal, 1991) and in problem solving. For example, complex physical systems, such as thermodynamic systems, often call for a deduction that must be made across several possible states of affairs. Hence, although our studies used double disjunctions, we expect that the underlying principles will apply to any sort of reasoning. On the one hand, it is difficult to consider several alternative states of affairs. On the other hand, any procedure (such as the use of an appropriate diagram) that helps individuals to keep track of possibilities will improve reasoning.

We draw three main conclusions. First, difficulty in thinking increases with disjunctive possibilities. Second, this difficulty can be ameliorated by the use of an appropriate diagram that helps reasoners to make explicit all the possibilities. Thus, certain diagrams can help individuals to reason more rapidly and more accurately. Third, these two phenomena vindicate the theory of mental models because, unlike other theories of reasoning and other accounts of the effects of diagrams, it predicts both of them. Logically untrained individuals tend to reason by constructing models of the situations described or depicted in the premises, and the effect of diagrams on the process is the first practical application of the theory of mental models. Finally, although we would like to claim to have discovered that diagrams can improve disjunctive reasoning, we note that Simon (1991, p. 96) reported anecdotally that engineers understood Supreme Court cases better when he represented them using circuit diagrams in which the switch positions corresponded to the yes/no decisions of the court.

REFERENCES

Barwise, J., & Etchemendy, J. (1992). Hyperproof: Logical reasoning with diagrams. In N.H. Narayanan (Ed.), *AAAI Spring Symposium on Reasoning With Diagrammatic Representations* (pp. 80–84). Unpublished manuscript.

Baylor, G.W. (1971). Programs and protocol analysis on a mental imagery task. In *First International Joint Conference on Artificial Intelligence*. Unpublished manuscript.

Beth, E.W. (1971). *Aspects of modern logic*. Dordrecht, The Netherlands: Reidel.

Braine, M.D.S. (1978). On the relation between the natural logic of reasoning and standard logic. *Psychological Review, 85*, 1–21.

Braine, M.D.S., Reiser, B.J., & Rumain, B. (1984). Some empirical justification for a theory of natural propositional logic. In G.H. Bower (Ed.), *The psychology of learning and motivation: Vol. 18* (pp. 313–371). San Diego: Academic Press.

Bruner, J.S., Goodnow, J.J., & Austin, G. (1956). *A study of thinking*. New York: Wiley.

Inhelder, B., & Piaget, J. (1958). *The growth of logical thinking from childhood to adolescence*. London: Routledge & Kegan Paul.

Johnson-Laird, P.N. (1983). *Mental models: Towards a cognitive science of language, inference, and consciousness*. Cambridge, MA: Harvard University Press.

Johnson-Laird, P.N., & Byrne, R.M.J. (1991). *Deduction*. Hillsdale, NJ: Erlbaum.

Johnson-Laird, P.N., Byrne, R.M.J., & Schaeken, W. (1992). Propositional reasoning by model. *Psychological Review, 33*, 418–439.

Larkin, J., & Simon, H. (1987). Why a diagram is (sometimes) worth 10,000 words. *Cognitive Science, 11*, 65–99.

Neisser, U., & Weene, P. (1962). Hierarchies in concept attainment. *Journal of Experimental Psychology, 64*, 640–645.

Osherson, D.N. (1974–1976). *Logical abilities in children* (Vols. 1–4). Hillsdale, NJ: Erlbaum.

Palmer, S.E. (1975). Visual perception and world knowledge: Notes on a model of sensory-cognitive interaction. In D.A. Norman, D.E. Rumelhart, & the LNR Research Group, *Explorations in cognition* (pp. 279–307). San Francisco: Freeman.

Pylyshyn, Z. (1973). What the mind's eye tells the mind's brain: A critique of mental imagery. *Psychological Bulletin, 80*, 1–24.

Rips, L.J. (1983). Cognitive processes in propositional reasoning. *Psychological Review, 90*, 38–71.

Shafir, E., & Tversky, A. (in press). Thinking through uncertainty: Nonconsequential reasoning and choice. *Cognitive Psychology*.

Simon, H.A. (1991). *Models of my life*. New York: Basic Books.

Smith, E.E., Langston, C., & Nisbett, R.E. (1992). The case for rules in reasoning. *Cognitive Science, 16*, 1–40.

Tabachneck, H., & Simon, H. (1992). Effect of mode of presentation on reasoning about economic markets. In N.H. Narayanan (Ed.), *AAAI Spring Symposium on Reasoning With Diagrammatic Representations* (pp. 56–64). Unpublished manuscript.

Tversky, A., & Shafir, E. (in press). The disjunction effect in choice under uncertainty. *Cognitive Science*.

Voss, J.F., Perkins, D.N., & Segal, J.W. (Eds.). (1991). *Informal reasoning and education*. Hillsdale, NJ: Erlbaum.

(Received 10/15/92; Revision accepted 3/9/93)